

Two Strategies for Handling Unknown Loads of Two Coordinating Robots

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In real situations, a robotic system can work in an unstructured environment in which the load is often unknown. This problem is an under-studied one, especially for multi-robot systems. In this paper we solve this problem by the 'Unknown Load Distribution' method for two coordinated industrial robots. Two methods are proposed for the distribution of an unknown load. The first method is called 'load estimated method', in which the parameters associated with the load are estimated using the information provided by two wrist force sensors. As a result, the load becomes known, and conventional optimal load distribution methods can then be applied to distribute the force. The second method is called the 'force compensation method', in which one of the robots (the leader) takes the major role of carrying the load to the exact location and the other robot (the follower) follows the leader with a specified force. The load is compensated by the follower using force control until the leader can carry the load to follow a satisfactory trajectory. To verify the force compensation method, a computer simulation is conducted.

Key Words: Unknown Load, Load Distribution, Coordinating Robot, Load Estimation Method, Force Compensation Method, Robot Parameter, Load Parameter, Compensating Torque, Compensating Force.

1. Introduction

The load distribution problem was previously studied by a number of papers for multiple manipulators (Zheng and Luh, 1988; Luh and Zheng, 1988; Cheng and Olin, 1989; Olim and Oh, 1981; Kreutz and Lokshin, 1988; Kumar and Waldron, 1988; Hsu, 1989; Walker et al., 1989; Pittelkau, 1988; Wang and Rami, 1988; Shin and Makay, 1987; Walker et al., 1989; Yoshikawa and Sudou, 1990) and multi-finger hands (Demmel and Lafferriere, 1989; Park and Starr, 1989; Cole et al., 1989; Murray and Sastry, 1989; Li et al., 1989; Kerr and Roth, 1986; Arimoto et al., 1987; Yoshikawa and Nagai, 1987). In general, these studies address the problem of how a load should be optimally distributed among multiple robots or multiple fingers such that certain criterion can be met. For example, Orin and Oh (1981) have

studied the control of force distribution for closed-chain robotic mechanisms. A weighed combination of energy consumption and load balancing was selected as a criterion. The linear programming technique used to obtain a solution. More recently, Zheng and Luh (1988) proposed several distribution methods for two coordinated manipulators using a number of criteria, including least energy consumption, minimum force exertion on the end-effectors, etc.

In most of the previous studies, the load mass and inertia are assumed known. In reality, however, when a robotic system works in an unstructured environment, the load is often unknown. Unknown load distribution is an under-studied problem, especially for multiple robot system. Shin and McKay (1987) address the robust trajectory-planning problem for a single manipulator under payload uncertainty. Walker et al. (1989) propose the adaptive coordinated motion control of two arms with unknown load

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mass. Yoshikawa and Sudou (1990) deals with the on-line estimation of unknown constraints. Atkeson et al. (1986) estimated the load parameters in the one arm case using a wrist force sensor. In this paper, the unknown load problem for two coordinated arms is analyzed and simulated.

We propose two methods for load distribution among two industrial robots. The first method is called 'load estimation method' and the second method is called 'force compensation method'. In the load estimation method, the load is estimated by using two wrist force sensors installed in each robot. Once the load is estimated, then all the distribution methods previously developed in the other works can be used. Our method will be an extension of the methods developed for load estimation of a single manipulator as studied in (Atkeson et al., 1986).

The force compensation method is a completely new approach. In this approach, one of the robots (the leader) takes the major responsibility of carrying the load. Only when the load cannot be sustained by the leader, the other robot (the follower) assist. Since the load does not need to be estimated before the load starts to be carried, this method is more responsive and computationally efficient than the first method.

The origination of the paper is as follows. In Section 2, load estimation by two industrial robots is first studied. In Section 3, our study is concentrated on the force compensation method. Simulation results for the force compensation method are presented in the Section 4, followed by the conclusions.

2. Load Estimation Method

Load estimation was previously studied for a single robot arm in (Atkeson et al., 1986), in which a wrist force sensor was used to estimate the force. In our study, the estimation method of (Atkeson et al., 1986) is extended to two arms holding one rigid object. In this case, two wrist force sensors are needed for the estimation of the load. For convenience, robot 1 is labelled as the leader, and robot 2 as the follower. We first derive the Newton-Euler equations for the two robots

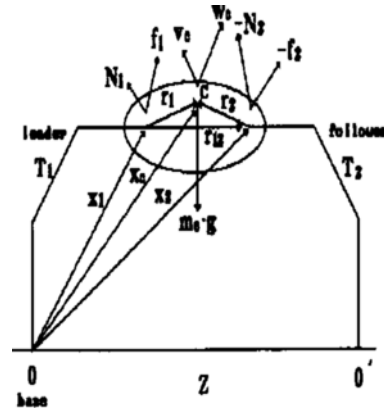


Fig. 1 Free body diagram of the load when two robots hold one object. (v_c is the linear velocity at the centroid, i.e., time derivative of x_c .)

holding one rigid object (Fig. 1). From Fig. 1, it can be seen that Newton's equation for the load is

$$f_1 - f_2 + m_c \cdot g - m_c \cdot \ddot{x}_c = 0 \quad (1)$$

where f_1 : force exerted at the leader end-effector,
 f_2 : force exerted at the follower end-effector,

m_c : mass of the load,

\ddot{x}_c : acceleration at the centroid of the load,

c : mass center (centroid) of the load and

g : gravitational acceleration.

Also from Fig. 1, the Euler equation of the load is

$$N_1 - N_2 - (r_{12} + r_2) \underline{x} f_1 + r_2 \underline{x} f_2 - I_c \cdot \dot{w}_c - w_c \underline{x} (I_c \cdot w_c) = 0 \quad (2)$$

where N_1, N_2 are the moments of the leader and follower end-effectors, respectively,

r_{12}, r_2 are the position vector from leader to follower and follower to centroid, respectively,

I_c : the centroidal inertia tensor of load,

w_c, \dot{w}_c are the rotational velocity and the acceleration of the load, respectively and

\underline{x} is the vector product.

As for the kinematics, the following transformation relation can be obtained from Fig. 1:

$$X = T_1^{-1} \cdot Z \cdot T_2 \quad (3)$$

where T_1, T_2 are 4×4 homogeneous transforma-

tion matrices from the base to the end-effectors of the leader and follower, respectively,

Z is the transformation matrix from the end-effector of the leader to the end-effector of the follower (4×4 matrix),

X is the transformation matrix from the end-effector of the leader to the end-effector of the follower (4×4 matrix).

Let the rotational matrix (3×3) of X be R , and the position vector (3×1) of X be P . Then, P is the same as r_{12} in Fig. 1, i.e.,

$$X = \begin{bmatrix} R & P \\ 000 & 1 \end{bmatrix}, \text{ where } P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = r_{12} \quad (4)$$

And let the rotation part of T_1 be R_1 , and the position part be P_1 , namely

$$P_1 = \begin{bmatrix} R_1 & P_1 \\ 000 & 1 \end{bmatrix} \quad (5)$$

Then we have

$$R_1 \cdot r_{12} = x_1 - x_2 \quad (6)$$

where x_1, x_2 are position vectors from the base to the end-effector of the leader and follower, respectively. Based on these dynamic and kinematic relation, the leader and follower dynamics can be combined as below. Let f and N be

$$f = f_1 - f_2 \text{ and} \quad (7)$$

$$N = N_1 - N_2 - r_{12} \underline{x} f_1. \quad (8)$$

Replacing f_1, f_2, N_1 and N_2 in (1) and (2) by (7) and (8), one obtains

$$f = -m_c \cdot g + m_c \cdot \dot{x}_c \text{ and} \quad (9)$$

$$N = r_2 \underline{x} f_1 - r_2 \underline{x} f_2 + I_c \cdot \dot{w}_c + w_c \underline{x} (I_c \cdot w_c) \\ = r_2 \underline{x} f + I_c \cdot \dot{w}_c + w_c \underline{x} (I_c \cdot w_c). \quad (10)$$

Meanwhile, from Fig. 1, one has

$$x_c = x_2 + r_2, \quad (11)$$

and the first and second time derivatives of (11) are

$$\dot{x}_c = \dot{x}_2 + w_c \underline{x} r_2 \text{ and} \quad (12)$$

$$\ddot{x}_c = \ddot{x}_2 + \dot{w}_c \underline{x} r_2 + w_c \underline{x} (w_c \underline{x} r_2). \quad (13)$$

Here, $\dot{w}_c \underline{x} r_2$ can vanish, if the angular velocity of the rotation about fixed axes is constant. By using

Eq. (13), Eq. (9) becomes

$$f = m_c \cdot (-g + \dot{x}_2) + \dot{w}_c \underline{x} m_c \cdot r_2 \\ + w_c \underline{x} (w_c \underline{x} m_c \cdot r_2). \quad (14)$$

using Eq. (14), Eq. (10) becomes

$$N = r_2 \underline{x} m_c \cdot (-g + \dot{x}_2) + r_2 (\dot{w}_c \underline{x} m_c \cdot r_2) \\ + r_2 \underline{x} [w_c \underline{x} (w_c \underline{x} m_c \cdot r_2)] + I_c \cdot \dot{w}_c \\ + w_c \underline{x} (I_c \cdot w_c) = (g - \dot{x}_2) \underline{x} m_c \cdot r_2 \\ + m_c \cdot r_2 \underline{x} (\dot{w}_c \underline{x} r_2) + m_c \cdot r_2 \underline{x} [w_c \underline{x} (w_c \underline{x} r_2)] \\ + I_c \cdot \dot{w}_c + w_c \underline{x} (I_c \cdot w_c). \quad (15)$$

From the three dimensional version of the Parallel Axis Theorem as cited in (Symon, 1971), one has

$$I_2 = I_c + m_c [(r_2^T \cdot r_2) \cdot I - (r_2 \cdot r_2^T)] \quad (16)$$

where I : a 3×3 identity matrix and

I_2 : inertia matrix with respect to the follower end-effector, i. e.,

$$I_2 = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}, \text{ a symmetric matrix.}$$

Using Eq. (16) and with some manipulations, (15) becomes

$$N = (g - \dot{x}_2) \underline{x} m_c \cdot r_2 + I_c \cdot \dot{w}_c + m_c [(r_2^T \cdot r_2) \cdot I \\ - (r_2 \cdot r_2^T)] \dot{w}_c + w_c \underline{x} \{ I_c + m_c [(r_2^T \cdot r_2) \cdot I \\ - (r_2 \cdot r_2^T)] \} w_c = (g - \dot{x}_2) \underline{x} m_c \cdot r_2 + I_2 \cdot \dot{w}_c \\ + w_c \underline{x} (I_2 \cdot w_c) \quad (17)$$

where $I_2 \cdot w_c = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \dot{w}'_c \cdot I'_2$, in which

$$\dot{w}'_c = \begin{bmatrix} w_x & w_y & w_z & 0 & 0 & 0 \\ 0 & w_x & 0 & w_y & w_z & 0 \\ 0 & 0 & w_x & 0 & w_y & w_z \end{bmatrix} \text{ and}$$

$$I'_2 = [I_{11} \ I_{12} \ I_{13} \ I_{22} \ I_{23} \ I_{33}]^T.$$

Rearranging Eqs. (14) and (17) into matrix form, one obtains

$$\begin{bmatrix} f \\ N \end{bmatrix} = \begin{bmatrix} \dot{x}_2 - g & [w_c \underline{x}] \\ 0 & [(g - \dot{x}_2) \underline{x}] \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_c \\ m_c \cdot r_2 \\ I'_2 \end{bmatrix} \begin{bmatrix} w_c \\ w'_c \end{bmatrix} \quad (18)$$

Denote the above equation as

$$F_{ii} = W_i \cdot L_i \quad (19)$$

where $F_{ii} = \begin{bmatrix} f \\ N \end{bmatrix}$, a 6×1 vector,

$$W_i = \begin{bmatrix} \dot{x}_2 - g & [\dot{w}_c x] + [w_c x] [w_c x] \\ 0 & [(g - \dot{x}_2) x] \\ & 0 \\ & [\cdot \dot{w}_c] + [w_c x] [\cdot w_c] \end{bmatrix} \text{ and}$$

$$L_i = \begin{bmatrix} m_c \\ m_c \cdot r_2 \\ I_2 \end{bmatrix}, \text{ a } 10 \times 1 \text{ vector.}$$

Here there are 6 known variables (3 in f , and the other 3 in N), and 10 unknown variables (1 in m_c , 3 in r_2 , and the other 6 in I_2). Eq. (19) therefore represents 6 equations in 10 unknown variables. Consequently, we need at least two different manipulator configurations to solve the unknown variables (12 equations, 10 unknowns). However, in general, more data sets can make the estimation more precise. If n data sets are used ($n \geq 2$), the least-square estimation yields

$$W^T \cdot F_t = W^T \cdot W \cdot L_i \quad (20)$$

from which one has

$$L_i = (W^T \cdot W)^{-1} \cdot W^T \cdot F_t \quad (21)$$

where $W = \begin{bmatrix} W_1 \\ \vdots \\ W_n \end{bmatrix}$, and $F_t = \begin{bmatrix} F_{t1} \\ \vdots \\ F_{tn} \end{bmatrix}$.

From Eq. (21), it can be seen that mass (m_c) and moment of inertia (I_2) as well as r_2 can be calculated as long as F_t is provided. Using r_2 , r_1 can be further derived as

$$r_1 = r_{12} - r_2 \quad (22)$$

where r_{12} was obtained by using Eq. (6). Using I_2 and r_2 , however, I_c can be estimated using (16). With the estimated load, the unknown load distribution can be treated as a known load distribution problem. For a known load, existing methods previously developed by other researchers (Zheng and Luh, 1988; Luh and Zheng, 1988; Cheng and Olin, 1989; Olin and Oh, 1981; Kreutz and Lekshin, 1988; Kumar and Waldron, 1988; Hsu, 1989; Walker et al., 1989; Pittelkau, 1988; Wong and Ravn, 1988; Shin and Makay, 1987) can now be used.

3. Force Compensation Method

In the force compensation method, the load is

not estimated. Instead, the leader takes the main role of carrying the load, and use the position control strategy to move the object along a pre-defined motion profile. Only when the trajectory of the object cannot satisfy the pre-defined motion profile, the follower starts to help. In this regard, the motion errors of the object are transformed into the required compensating force for the follower. the follower then uses force control to provide the compensating force. As a result, the position force control strategy as developed in our previous work can be used (Kim and Zheng 1989). In Kim and Zheng (1989), it also has been proved that this kind of position force control strategy is stable for two industrial robots carrying a single rigid object. To apply this position force control strategy, a wrist force sensor is required to be installed on the follower (Kim and Zheng 1989).

3.1 Strategy and magnitude of force compensation

When the leader follows a given trajectory with admissible error bounds, the follower just follows the leader without providing any force. When the leader is overburdened, the position error is beyond a certain threshold. This means that one or more of the joint torques of the leader exceed the maximal joint torque, i. e.,

$$|T_{1i}| \geq |T_{1imax}| \quad (23)$$

where T_{1i} : i -th joint torque of leader and T_{1imax} : the maximum limit of T_{1i} .

In this case, the position error becomes

$$|x_1 - x_{id}| = \varepsilon \geq \delta \quad (24)$$

where ε : position error and

δ : admissible position error bound.

In position control, there always exists some position error in normal control, and this kind of position error is usually within a bound δ . However, when the load is too heavy, the position error ε will be greater than the bound δ . The problem is how to reduce the absolute value of the required torque at joint i . Unfortunately, the torque of the leader is not measured, because the leader uses position control. The only parameter

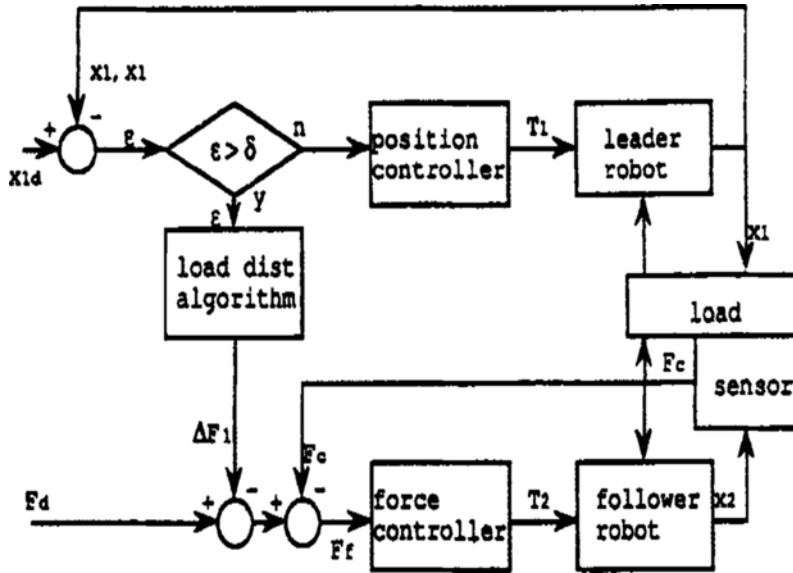


Fig. 2 Block diagram for unknown load distribution. (force compensation method)

that can be used is the actual position of the joint. One possible way of reducing the leader torque is to reduce the load of the leader. The overall control strategy of the system is as follows (Fig. 2):

First, the position error is changed to a torque value (ΔT_1) for the leader using

$$|\Delta T_1| = |J_1^T \cdot k_p \cdot \varepsilon| \quad (25)$$

where ΔT_1 : required compensating torque,

J_1^T : transpose of leader Jacobian matrix and

k_p : position feedback gain.

Here only the magnitude is selected, and the direction of the compensating torque will be determined in the next section.

Secondly, after the required compensating torque is determined, the compensating force of the leader can be calculated as

$$\Delta F_1 = (J_1^T)^{-1} \cdot \Delta T_1. \quad (26)$$

This compensating force (ΔF_1) should be provided by the follower. This means that the compensating force should be added to the follower force that is originally designed for the follower end-effector. From Fig. 2, it can be seen that

$$F_f = F_d - (\Delta F_1 + F_c) \quad (27)$$

where F_f : input force of the force controller.

F_d : desired force of the follower and

F_c : reaction force at the sensor.

Here, $F_c + \Delta F_1$ is considered as the new follower reaction force. Since, in real applications a force sensor is attached to the follower wrist, $F_c = F_2$. Thus the effect of the compensation is equivalent to the change of the follower reaction force as follows:

$$F'_2 = F_2 + \Delta F_1 \quad (28)$$

where F'_2 is the new follower end-effector force after compensation. This equation gives the new follower force which needs to be provided to the object. The force also reflects the compensation effect to the leader.

This change of force is now transferred to the leader using force control of the follower (Shin and Makay, 1987). That is, find how the change of the follower end-effector force affects the force exerted on the leader. After the compensation, the force at the leader becomes (Fig. 3)

$$F'_1 = F_0 - F'_2 = F_1 - \Delta F_1 \quad (29)$$

where $F_0 = F_1 + F_2$ and F'_1 is the new leader end-effector force after compensation. The new end-effector force of the leader should result in smaller joint torques that need to be provided by the

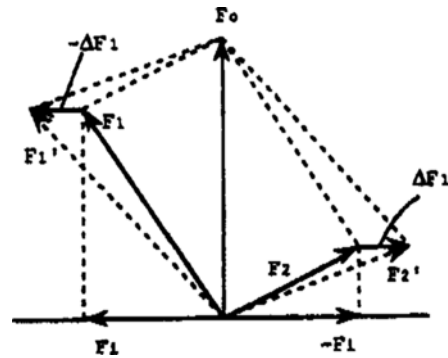
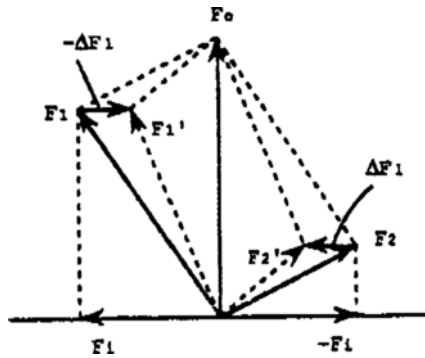


Fig. 3 Changing of the end-effector forces after force compensation.

leader. As a result, the position error of the object can be reduced. In order to do so, the direction of the compensating torque has to be determined. The reason for using the compensating torque instead of the compensating force is that the analysis of the selection of the direction is performed in the torque domain. Once the direction of the compensating torque is selected, the direction of the compensating force is determined by (26). In the next subsection, the direction of the compensating torque will be studied.

3.2 Direction of the compensating torque

In order to define the direction of the compensating torque, the robot dynamic equations are required. The simplified closed form dynamic equations of a robot (Tarn et al., 1985) are as follows:

$$D(\ddot{q})q + H(q, \dot{q})\dot{q} + G(q) = T + J^T F \quad (30)$$

where $D(q)$: mass matrix,
 $H(q, \dot{q})$: coriolis and centrifugal matrix,
 $G(q)$: gravitational vector,
 $q = [q_1 \dots q_n]^T$,
 \dot{q}, \ddot{q} are the first and second time derivative vectors of q , respectively,
 T : joint torque vector,
 F : external force vector at the end-effector, and
 J^T : transpose of the Jacobian matrix.

In (30), the elements of the left side are all known; let this part be denoted B . As a result, the leader and follower dynamic equations become

$$T_1 = B_1 - J_1^T \cdot F_1 \quad \text{and} \quad (31)$$

$$T_2 = B_2 - J_2^T \cdot F_2 \quad (32)$$

The load dynamic Eqs. (1) and (2) can be rewritten as follows :

$$\begin{bmatrix} M & 0 \\ 0 & I_c \end{bmatrix} \begin{bmatrix} \ddot{x}_c \\ \dot{w}_c \end{bmatrix} + \begin{bmatrix} 0 \\ w \times I_c \cdot w \end{bmatrix} + \begin{bmatrix} M \cdot G \\ 0 \end{bmatrix} = F_1 + F_2 \quad (33)$$

where M, I_c : 3×3 diagonal matrix representing the mass and moments of inertia of the load, respectively,

$$G = [0 \ 0 \ g]^T,$$

$$F_1 = \begin{bmatrix} f_1 \\ N_1 - (r_{12} + r_2) \times f_1 \end{bmatrix}, \text{ and}$$

$$F_2 = - \begin{bmatrix} f_2 \\ N_2 - r_2 \times f_2 \end{bmatrix}.$$

The elements of the left side of Eq. (33) are all known values once the motion of the object is defined. Let this part be denoted as F_0 . It follows that

$$F_0 = F_1 + F_2. \quad (34)$$

From Eq. (31), in order to reduce the absolute value of joint torque (T_1), both the robot dynamics (B_1) and end effector force (F_1) are involved. In the above three Eqs. (31), (32) and (34), the values of B_1, B_2, J_1, J_2 , and F_2 are known, and the values of T_1, T_2, F_1 , and F_0 are unknown. However, the direction of F_0 is known, because the trajectory of the object is given:

$$F_0 = K_0 F_{ou} \quad (35)$$

where F_{ou} : unit vector of F_0 , a known value and

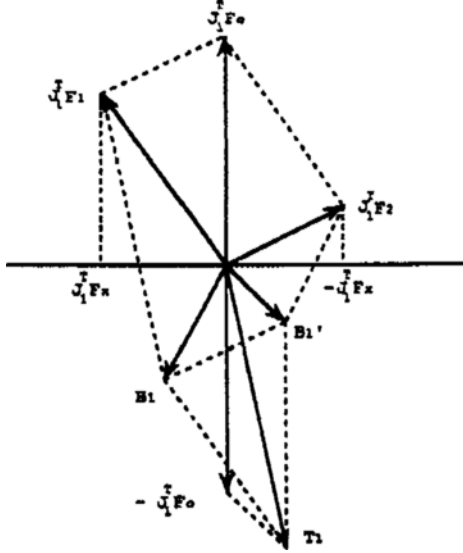


Fig. 4 Components in the torque domain.

K_0 : magnitude of F_0 , an unknown value.

Transfer this force value to the torque value using Jacobian, that is,

$$J_1^T \cdot F_0 = J_1^T \cdot (K_0 F_{ou}) = K_0 (J_1^T \cdot F_{ou}) \quad (36)$$

Since $(J_1^T \cdot F_{ou})$ is not a unit vector, we define the unit vector as $(J_1^T \cdot F_{ou})_u$. Then Eq. (36) becomes

$$J_1^T \cdot F_0 = K_0 K_0' (J_1^T \cdot F_{ou})_u = K' (J_1^T \cdot F_{ou})_u \quad (37)$$

where $(J_1^T \cdot F_{ou})_u = K_0' (J_1^T \cdot F_{ou})_u$ and

$$K' = K_0 K_0', \text{ an unknown value.}$$

Here, define the orthonormal of $(J_1^T \cdot F_{ou})_u$ as $(J_1^T \cdot F_x)_u$, for future use. From the previous subsection, the amount of compensating torque is already calculated as $|\Delta T_1|$ from Eq. (25), but the direction of the compensating torque is not specified yet. Therefore, from Eq. (26), the value of the end-effector compensating vector ΔF_1 is unknown. Meanwhile, ΔF_1 is changed to F_1' using the compensating value ΔF_1 as in Eq. (29). The torque at the joints of the leader then becomes (Fig. 4)

$$\begin{aligned} T_1' &= B_1 - J_1^T \cdot F_1' \\ &= B_1 - J_1^T \cdot (F_1 - \Delta F_1) \\ &= B_1 - J_1^T \cdot (F_0 - F_2) + J_1^T \cdot \Delta F_1 \quad (38) \\ &= (B_1 + J_1^T \cdot F_2) - J_1^T \cdot F_0 + J_1^T \cdot \Delta F_1 \\ &= B_1' - K' (J_1^T \cdot F_{ou})_u + \Delta T_1 \end{aligned}$$

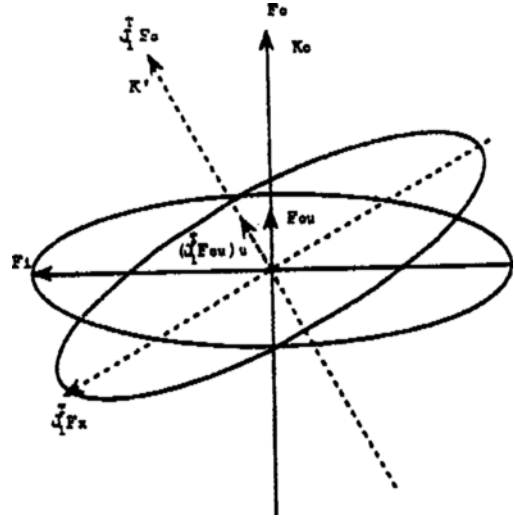


Fig. 5 Comparison of force and torque domain. F_{ou} is unit vector in the force domain. $(J_1^T F_{ou})_u$ is unit vector in the torque domain.

where T_1' is the leader joint torque changed by the compensating force and

$$B_1' = B_1 + J_1^T \cdot F_2 \quad (39)$$

which is a known vector.

Here, the original torque value at the leader joints is

$$T_1 = B_1' - K' (J_1^T \cdot F_{ou})_u \quad (40)$$

Therefore, Eq. (38) can be written as

$$T_1' = T_1 + \Delta T_1 \quad (41)$$

where ΔT_1 is the compensating torque. The values in Eq. (38) are partitioned into a component of $(J_1^T \cdot F_{ou})_u$ and the orthogonal component of $(J_1^T \cdot F_{ou})_u$, as $(J_1^T \cdot F_x)_u$. Here, $J_1^T \cdot F_{ou}$ is the value which reflects the torque at every joint of the leader. If this torque value is transformed into the force domain, it acts as a force which contributes to the motion of the object. However, if $J_1^T \cdot F_x$ is transformed into the force domain, it does not necessarily act as an internal force only, because the transformation matrix (Jacobian) is not linear (Fig. 5).

Now, B_1' in (39) can also be divided into the two orthogonal torque factors defined above. This is possible from Eqs. (31) and (39), in which B_1 , $J_1^T \cdot F_1$ and $J_1^T \cdot F_2$ all consist of two orthogonal vectors defined by $(J_1^T \cdot F_{ou})$ and $(J_1^T \cdot$

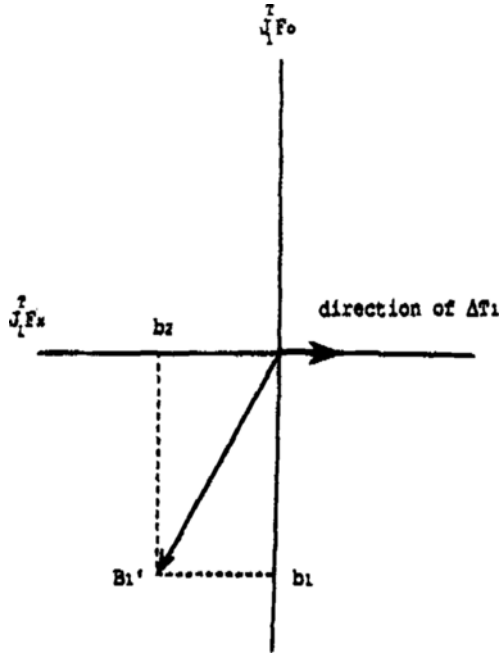


Fig. 6 Direction of ΔT_1 (Eq. 46)

F_x). Now, B_1 can be divided as (Fig. 6)

$$B_1' = b_1(J_1^T \cdot F_{ou})_u + b_2(J_1^T \cdot F_x)_u \quad (42)$$

where $b_1 = B_1'(J_1^T \cdot F_{ou})_u$,

$$b_2 = |B_1' - b_1(J_1^T \cdot F_{ou})_u| \text{ and}$$

$$(J_1^T \cdot F_x)_u = \frac{B_1' - b_1(J_1^T \cdot F_{ou})_u}{b_2}$$

ΔT_1 is divided as

$$\Delta T_1 = K_1(J_1^T \cdot F_{ou})_u + K_2(J_1^T \cdot F_x)_u \quad (43)$$

where K_1, K_2 are unknown constants. By using Eqs. (42) and (43), Eq. (38) becomes

$$T_1' = (b_1 - K_1 + K_1)(J_1^T \cdot F_{ou})_u + (b_2 + K_2)(J_1^T \cdot F_x)_u \quad (44)$$

From Eq. (44), one may find that the first term is for the motion of the robot only, so compensation has to be applied to eliminate the second component. In this application, compensation depends on the leader position error. The best possible compensation is

$$K_1 = 0 \text{ and } K_2 = -\text{sing}(b_2)|\Delta T_1| \quad (45)$$

where $\text{sing}(b_2)$ takes the sign of b_2 and the absolute value of $|\Delta T_1|$ comes from Eq. (25).

Now, the compensating torque is defined by Eqs. (43) and (45), as (Fig. 7)

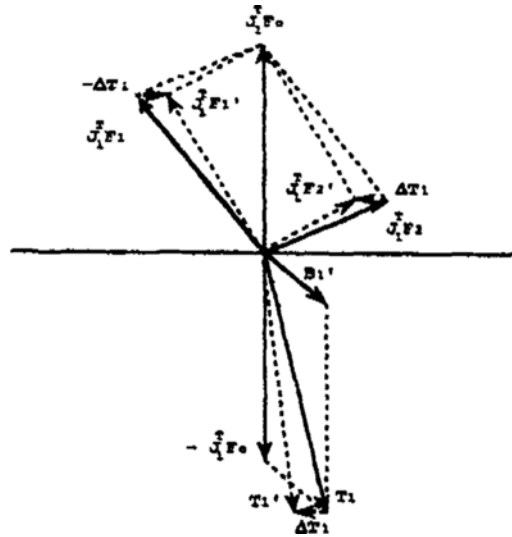


Fig. 7 Representation of compensating torque in the torque domain.

$$\Delta T_1 = -\text{sing}(b_2)|\Delta T_1|(J_1^T \cdot F_x)_u \quad (46)$$

Using Eq. (26), the compensating force ΔF_1 can also be determined, and the leader and follower end-effector forces in Eqs. (29) and (28) are changed. As a result, by the equations given above, the magnitude of the leader joint torque (T_1) will be decreased. Consequently, the position error of the object can be reduced. In the next section, simulation for the force compensation method will be studied.

4. Simulation and Results

To verify the force compensation method as proposed in the previous section, a computer simulation was conducted in our study. During the motion of two robots handling one heavy load, some required joint torques may be over their maximum limits. If this happens, the robot will not behave properly. For continuous motion of the robot, the load of the overloaded joint has to be reduced by load distribution. If we know the load, it is relatively simple. But in the unknown load case, it becomes more complicated as described in the previous sections.

For this simulation, all the position and joint torque parameters are adapted for the UNIMATION PUMA-560 robot. The control method

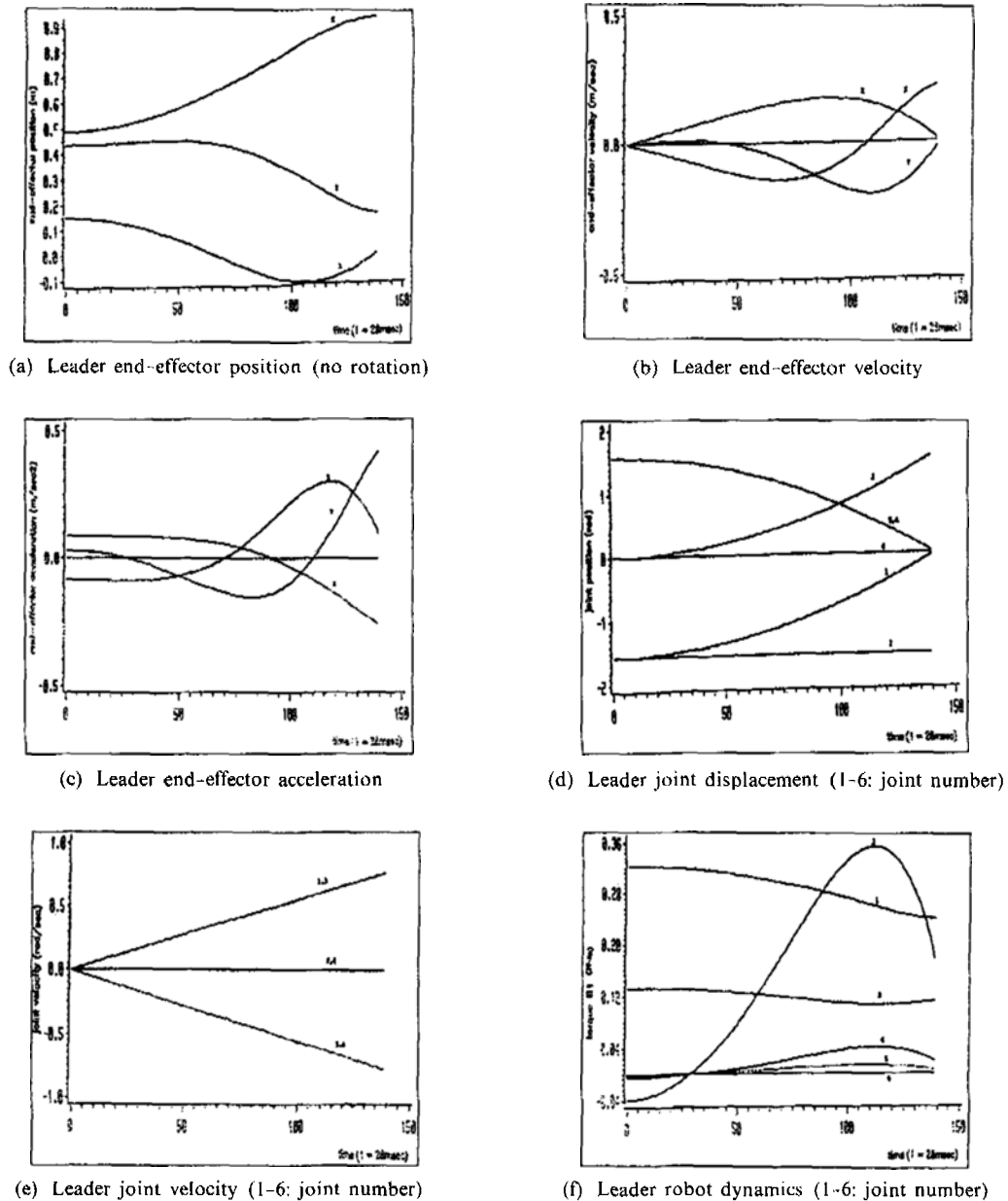


Fig. 8

used in this simulation are the same as in Kim and Zheng (1989), i. e., position and force control. For simplicity of the simulation, gravity terms are neglected.

4.1 Concepts and procedures of simulation

For normal operation, the leader follows the desired positions and the follower follows the desired forces given. If one of the leader joints is

overloaded by a heavy load, then the leader cannot follow the desired position. In this case, when the position error exceeds a certain threshold, the load distribution algorithm calculates the magnitude and direction of the compensating torque, and transforms this compensating torque into a compensating force. This compensating force is added to the follower end-effector force by force control. Because the sum of the leader

and the follower end-effector forces have to remain constant, the leader end-effector force is subtracted by the compensating force. As a result, the joint torque of the leader is reduced and the leader can move successfully along the desired trajectory. The follower can also follow the leader with the desired force.

4.2 Leader robot parameters

First, we assume that the approximate boundaries of the leader robot motion in joint coordinates are

$$q_i = [-90. \ -90. \ 0. \ 0. \ 90. \ 90.]^T \text{ and} \\ q_f = [0. \ -90. \ 90. \ 0. \ 0. \ 0.]^T. \quad (47)$$

where q_i is the initial position of the leader and q_f is the approximate final position of the leader.

Secondly, assume the joint accelerations to be

$$\ddot{q} = [.2 \ 0. \ .2 \ 0. \ -.2 \ -.2]^T \quad (48)$$

Thirdly, calculate the joint velocities and positions using the joint accelerations:

$$\dot{q} = \dot{q}_0 + \ddot{q} \cdot \text{delt} \quad (49)$$

where \dot{q}_0 is the velocity of the previous step and delt is the time interval of the step and

$$q = q_0 + \dot{q} \cdot \text{delt} + \ddot{q} \cdot (\text{delt}^2)/2 \quad (50)$$

where q_0 is the position of the previous step. The position is limited by the boundaries given Eq. (47). Figure 8 (d) and (e) represent joint displacement and velocity, respectively. Figure 8(d) represents the given motion trajectories of the leader joints.

Fourthly, using this joint parameters, calculate the end-effector position (x), velocity (\dot{x}), and acceleration (\ddot{x}) parameters. The end-effector position is determined by the kinematic relations of the robot, and the velocity and acceleration are determined by

$$R_0^6 \cdot \dot{x} = J \cdot \dot{q} \text{ and} \quad (51) \\ R_0^6 \cdot \ddot{x} = \dot{J} \cdot \dot{q} + J \cdot \ddot{q}.$$

where R_0^6 represents the rotation matrix from the base to the end-effector of the leader. Figure 8 (a), (b) and (c) represent the end-effector position, velocity, and acceleration, respectively. Figure 8 (a) represents the given motion trajectories

of the leader end-effector.

Fifthly, calculate the robot dynamics (B_1) using simplified the Newton-Euler equation as developed by Luh, Walker, and Paul for the PUMA 560 parameters (Luh et al., 1980; Tran et al., 1985). Figure 8 (f) represents as robot dynamics during the motion from (48) to (50).

4.3 Load parameters

In practice, we do not know the load parameters except the motion direction of the load. But for the verification of the load distribution, forces of the leader and the follower end-effectors are calculated as below.

Assume that the load configuration is as shown in Fig. 9, and the mass (m) is 8 kg (the maximum load of a single PUMA 560 in 4 kg). The acceleration of load (\ddot{x}) is calculated as in the previous subsection. By the mass and acceleration value of the load, the resultant force (F_0) can be calculated as

$$F_0 = m \cdot \ddot{x} \quad (52)$$

This is true when we neglect the gravitational term ($F_g = 0$, in Fig. 9), namely the resultant force is the same as the motion direction ($F_0 = F_r$, in Fig. 9). Also, by Eq. (34), end-effector forces (F_1, F_2) can be calculated using

$$F_0 = W \cdot \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (53)$$

where $W = [I_6, I_6]$, a 6×12 matrix, in which I_6 is

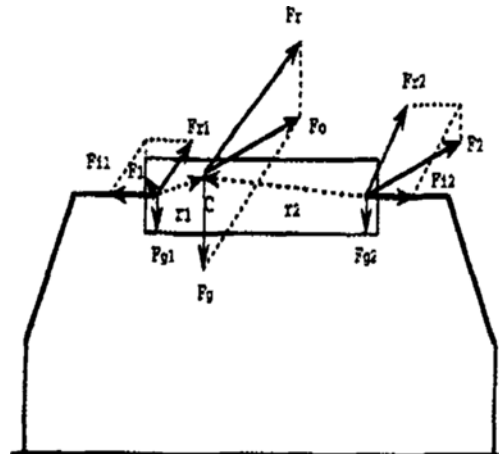


Fig. 9 Force components of load and end-effectors due to gravity and motion.

a 6×6 identity matrix. In Eq. (53), we can select W in many other ways. One alternate form is

$$W = \begin{bmatrix} I_3, 0, I_3, 0 \\ S_1, I_3, S_2, I_3 \end{bmatrix} \quad (54)$$

where $S_i = r_i \times I_3$,

I_3 is 3×3 identity matrix and

r_i is defined in Fig. 9, $i=1,2$.

But this method does not give any difference in the analysis if we choose the internal force vector of the inverse form of Eq. (53). For this simulation, W is selected as (54), and S_1 and S_2 are determined from Fig. 9 as

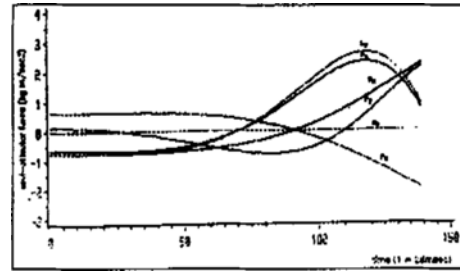
$$S_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 0 \\ -4 & 0 & 0 \end{bmatrix}. \quad (55)$$

Figure 10(a) and (b) represent end-effector forces of the leader and the follower, respectively, while Fig. 10(c) represents internal forces. Using these force values and the robot dynamics, we can calculate the leader joint torque. Figure 10(d) represents the leader joint torque before compensation.

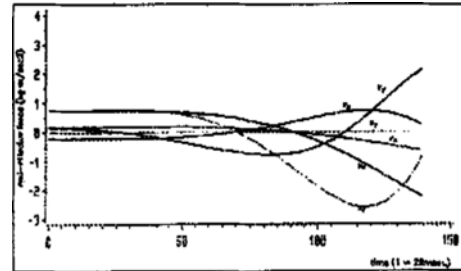
4.4 Results of load compensation

Because of the overload occurring at some of the joints, the position error will exceed the error bound. We first select the leader position error bound to be 0.004 inch (This is the position precision of PUMA 560.). From the given motion trajectory of leader end-effector (Fig. 8 (a)), the joint torque of joint 6 exceeds the limit of its joint torque (assume 3Nm of Fig. 10 (d)). This means the position error exceeds its limit as well. If the position error exceeds the limit, we calculate the amount of compensating torque by Eq. (25). Then we select the direction of compensating torque by Eq. (46). Finally, this compensating torque is transformed into the compensating force by Eq. (26).

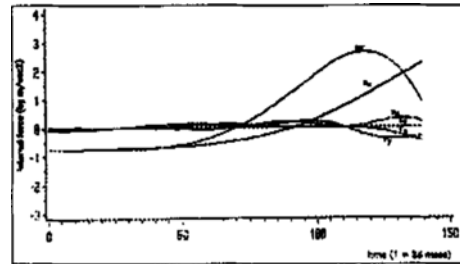
Add this compensating force to the follower end-effector force (F_2) by force control (Fig. 11 (a)). As the resultant force (F_0) remains the same, the leader end-effector force (F_1) will be subtracted by the compensating force (Fig. 11 (b)). Also the internal force (F_i) will be changed



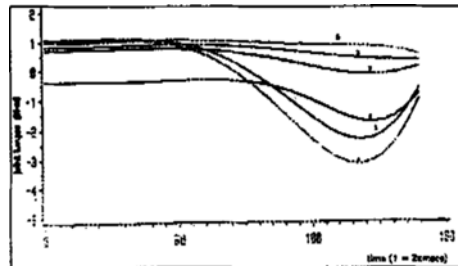
(a) Leader end-effector force before compensation



(b) Follower end-effector force before compensation



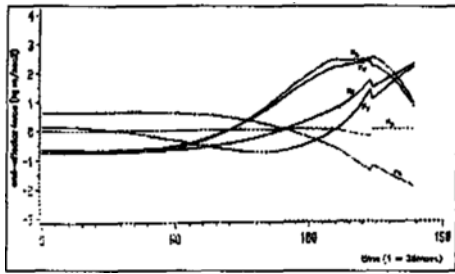
(c) Internal force before compensation



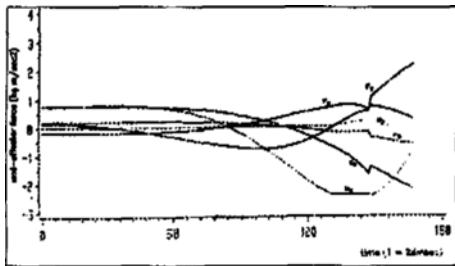
(d) Leader joint torque before compensation (1-6: Joint number)

Fig. 10

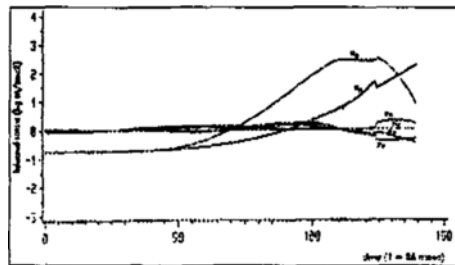
according to the variation of end-effector force (Fig. 11 (c)). This change of end-effector forces



(a) Leader end-effector force after compensation



(b) Follower end-effector force after compensation



(c) Internal force after compensation

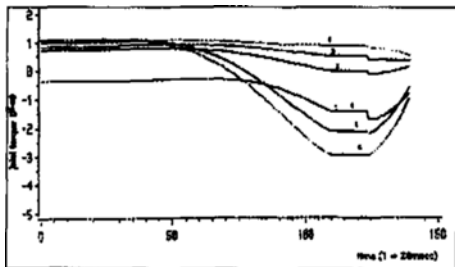
(d) Leader joint torque after compensation
(1-6: Joint number)

Fig. 11

affects the joint torque by Eq. (31) and (32). By

selecting the magnitude and direction of the compensating torque as in Eq. (46), the joint torque of the leader is decreased and never exceeds 3 Nm (Fig. 11(d)). This also guarantees the position error is within the admissible bound (0.004inch) by Eq. (25).

It is noted that when the compensation force is active, the magnitude of all the joint torques is changed. This is because the compensation actually provides a measure of compensation torque to all the joints, although the error is originally caused by only one joint (joint 6 in this simulation).

5. Conclusion

In this paper, the unknown load distribution problem is studied. Two different approaches are used to solve the problem of unknown load distribution. The first approach estimates the load using two wrist force sensors. previous methods for optimal load distribution can be applied.

The second approach provides force compensation by the follower when the leader is overloaded. In this method, the position error of the leader, which is caused by the compensating torque value will be reduced. The compensating force of the leader can be calculated by this compensating torque. This compensating force is provided by the follower using a force control strategy. As a result, the leader can follow the desired motion trajectory of the object. Simulation results are provided for the force compensation method to demonstrate its validity and performance.

References

Arimoto, S., Miyazaki, F. and Kawamura, S., 1987, "Cooperative Motion Control of Multiple Robot Arms or Fingers," *Proc. IEEE Intern. Conference on Robotics and Automation*, Raleigh, NC, pp. 1407~1412.

Asada, H. and Slotine, J. J. E., 1986, *Robot Analysis and Control*, A Wiley-Interscience Publication.

Atkeson, C. G., An, C. H., and Hollerbach, J.

- M., 1986, "Estimation of Inertial Parameters of Manipulator Loads and Links," *the Intern. J. of Robotics Research*, Vol. 5, No. 3, pp. 101~119.
- Cheng, F. T. and Olin, D. E., 1989, "Efficient Algorithm for Optimal Force Distribution in Multiple-Chain Robotic Systems : the Compact-Dual LP Method," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 934~950.
- Cole, A. A., Hsu, P. and Sastry, S., 1989, "Dynamic Regrasping by Coordinated Control of Sliding for a Multi-Fingered Hand," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 781~786.
- Demmel, J and Lafferriere, G., 1989, "Optimal Three Finger Grasps," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 936~942.
- Hsu, P., 1989, "Control of Multi-Manipulator Systems Trajectory Tracking, Load Distribution, Internal Force Control, and Decentralized Architecture," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 1234~1239.
- Kerr, J. and Roth, B., 1986, "Special Grasping Configurations with Dexterous Hands," *Proc. IEEE Intern. Conference on Robotics and Automation*, San Francisco, CA, pp. 1361~1367.
- Kim, K. I. and Zheng, Y. F., 1989, "Two Strategies of Position and Force Control for Two Industrial Robots Handling a Single Object," *Robotics and Autonomous Systems*, Vol. 5, No. 4, pp. 395~403.
- Kreutz, K. and Lokshin, A., 1988, "Load Balancing and Closed Chain Multiple Arm Control," *JPL Engineering Memo*, 347-88-233.
- Kumar, V. R. and Waldron, K. J., 1988, "Force Distribution in Closed Kinematic Chains," *IEEE J. of Robotics and Automation*, Vol. 4, No. 6, pp. 657~664.
- Li, Z., Canny, J. F. and Sastry, S. S., 1989, "ON Motion Planning for Dexterous Manipulation, Part I : The problem Formulation," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 775~780.
- Luh, J. Y. S. and Zheng, Y. F., 1988, "Load Distribution between Two Coordinating Robots by Nonlinear Programming," *Proc. ACC*, Atlanta, GA, pp. 479~482.
- Luh, J. Y. S., Walker, M. W. and Paul, R. P., 1980, "On-Line Computational Scheme for Mechanical Manipulators," *Trans. Of ASME, Journal of DSMC*, Vol. 102, pp. 69~76.
- Murray, R. M. and Sastry, S. S., 1989, "Control Experiments in Planner Manipulation and Grasping," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 624~629.
- Olin, D. E. and Oh, S. Y., 1981, "Control of Force Distribution in Robotic Mechanisms Closed Kinematic Chains," *Trans. Of ASME, J. of DSMC*, Vol. 102, pp. 134~141.
- Park, Y. C. and Starr, G. P., 1989, "Finger Force Computation for Manipulation of an Object by a Multi-fingered Robot Hand," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 930~935.
- Pittelkau, M. E., 1988, "Adaptive Load-Sharing Force Control for Two-Arm Manipulators," *IEEE Intern. Conference on Robotics and Automation*, Philadelphia, PA, pp. 498~503.
- Shin, K. G. and Makay, N. D., 1987, "Robust Trajectory Planning for Robotic Manipulators under Payload Uncertainties," *IEEE Trans. On Automatic Control*, Vol. AC-32, No. 12, pp. 1044~1054.
- Symon, K. R., *Mechanics*, Reading, Mass. : Addison-Wesley.
- Tarn, T. J., Bejczy, A. K. and Yun, X., 1985, "Dynamic Equations for Puma 560 Robot Arm," *Robotics Laboratory Report*, SSM-RL-85-02, Washington University.
- Walker, I. D., Freeman, R. A. and Marcus, S. I., 1989, "Internal Object Loading for Multiple Cooperating Robot Manipulators," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 606~611.
- Walker, I. D., Marcus, S. I. and Freeman, R. A., 1989, "Distribution of Dynamic Loads for Multiple Cooperating Robot Manipulators," *J. of Robotic Systems*, Vol. 6(1), pp. 35~47.
- Walker, M. W., Kim, D., and Dionise, J., 1989, "Adaptive Coordinated Motion Control of Two Manipulator Arms," *Proc. IEEE Intern. Conference on Robotics and Automation*, pp. 1084~1090.
- Wang, L. T and Ravani, B., 1988, "Dynamic

Load Carrying Capacity of Mechanical Manipulators-Part I : Problem Formulation, Part II : Computational Procedure and Applications," *Trans. Of ASME, Journal of DSMC*, Vol. 110, pp. 46~61.

Yoshikawa, T., and Sudou, A., 1990, "Dynamic Hybrid Position/Force Control of Robot Manipulators: On Line Estimation of Unknown Constraints," *Proc. Of IEEE Int. Conf. On Robotics and Automation*, pp. 1231~1236.

Yoshikawa, T. and Nagai, K., 1987, "Manipulating and Grasping Forces in Manipu-

lation by Multi-Fingered Hands," *Proc. IEEE Intern. Conference on Robotics and Automation*, Raleigh, NC, pp. 1998~2004.

Zheng, Y. F. and Luh, J. Y. S., 1986, "Joint Torques for Control of Two Coordinated Moving Robots," *Proc. IEEE Intern. Conference on Robotics and Automation*, San Francisco, CA, pp. 1375~1380.

Zheng, Y. F. and Luh, J. Y. S., 1988, "Optimal Load Distribution for Two Industrial Robots Handling a Single Object," *Proc. IEEE Intern. Conference on Robotics and Automation*, Philadelphia, PA, pp. 344~349.